Mapping Out $SU(5)$ GUTs with Non-Abelian Discrete Flavor Symmetries

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We construct a class of supersymmetric $SU(5)$ GUT models that produce nearly tribimaximal lepton mixing, the observed quark mixing matrix, and the quark and lepton masses, from discrete non-Abelian flavor symmetries. The $SU(5)$ GUTs are formulated on five-dimensional throats in the flat limit and the neutrino masses become small due to the type-I seesaw mechanism. The discrete non-Abelian flavor symmetries are given by semi-direct products of cyclic groups that are broken at the infrared branes at the tip of the throats. As a result, we obtain $SU(5)$ GUTs that provide a combined description of non-Abelian flavor symmetries and quark-lepton complementarity.

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One possibility to explore the physics of grand unified theories (GUTs) [1, 2] at low energies is to analyze the neutrino sector. This is due to the explanation of small neutrino masses via the seesaw mechanism [3, 4], which is naturally incorporated in GUTs. In fact, from the perspective of quark-lepton unification, it is interesting to study in GUTs the drastic differences between the masses and mixings of quarks and leptons as revealed by current neutrino oscillation data.

In recent years, there have been many attempts to reproduce a tribimaximal mixing form [5] for the leptonic Pontecorvo-Maki-Nakagawa-Sakata (PMNS) [6] mixing matrix U_{PMNS} using non-Abelian discrete flavor symmetries such as the tetrahedral [7] and double (or binary) tetrahedral [8] group

$$A_4 \simeq Z_3 \ltimes (Z_2 \times Z_2) \quad \text{and} \quad T' \simeq Z_2 \ltimes Q, \tag{1}$$

where Q is the quaternion group of order eight, or [9]

$$\Delta(27) \simeq Z_3 \ltimes (Z_3 \times Z_3), \tag{2}$$

which is a subgroup of $SU(3)$ (for reviews see, e.g., Ref. [10]). Existing models, however, have generally difficulties to predict also the observed fermion mass hierarchies as well as the Cabibbo-Kobayashi-Maskawa (CKM) quark mixing matrix V_{CKM} [11], which applies especially to GUTs (for very recent examples, see Ref. [12]). Another approach, on the other hand, is offered by the idea of quark-lepton complementarity (QLC), where the solar neutrino angle is a combination of maximal mixing and the Cabibbo angle θ_{C} [13]. Subsequently, this has, in an interpretation of QLC [14, 15], led to a machine-aided survey of several thousand lepton flavor models for nearly tribimaximal lepton mixing [16].

Here, we investigate the embedding of the models found in Ref. [16] into five-dimensional (5D) supersymmetric (SUSY) $SU(5)$ GUTs. The hierarchical pattern of quark and lepton masses, V_{CKM}, and nearly tribimaximal lepton mixing, arise from the local breaking of non-Abelian discrete flavor symmetries in the extra-dimensional geometry. This has the advantage that the

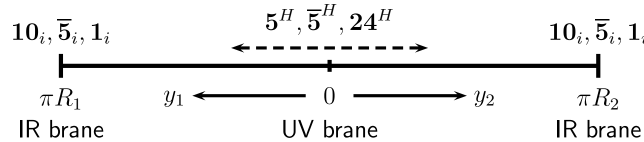

FIG. 1: SUSY $SU(5)$ GUT on two 5D intervals or throats. The zero modes of the matter fields $\mathbf{10}_i, \overline{\mathbf{5}}_i$, and $\mathbf{1}_i$, are symmetrically located at $y_1 = \pi R_1$ and $y_2 = \pi R_2$, whereas the Higgs hypermultiplets $\mathbf{5}^H, \overline{\mathbf{5}}^H, \mathbf{24}^H$, and the gauge supermultiplet, propagate freely in the two throats.

scalar sector of these models is extremely simple without the need for a vacuum alignment mechanism, while offering an intuitive geometrical interpretation of the non-Abelian flavor symmetries. As a consequence, we obtain, for the first time, a realization of non-Abelian flavor symmetries and QLC in $SU(5)$ GUTs.

We will describe our models by considering a specific minimal realization as an example. The main features of this example model, however, should be viewed as generic and representative for a large class of possible realizations. Our model is given by a SUSY $SU(5)$ GUT in 5D flat space, which is defined on two 5D intervals that have been glued together at a common endpoint. The geometry and the location of the 5D hypermultiplets in the model is depicted in FIG. 1. The two intervals constitute a simple example for a two-throat setup in the flat limit (see, e.g., Refs. [17, 18]), where the two 5D intervals, or throats, have the lengths πR_1 and πR_2, and the coordinates $y_1 \in [0, \pi R_1]$ and $y_2 \in [0, \pi R_2]$. The point at $y_1 = y_2 = 0$ is called ultraviolet (UV) brane, whereas the two endpoints at $y_1 = \pi R_1$ and $y_2 = \pi R_2$ will be referred to as infrared (IR) branes. The throats are supposed to be GUT-scale sized, i.e. $1/R_{1,2} \gtrsim M_{\text{GUT}} \simeq 10^{16}\,\text{GeV}$, and the $SU(5)$ gauge supermultiplet and the Higgs hypermultiplets $\mathbf{5}^H$ and $\overline{\mathbf{5}}^H$ propagate as bulk fields freely on the two intervals. In usual 5D GUT models, $SU(5)$ is broken to the standard model (SM) gauge group $G_{\text{SM}} = SU(3)_c \times SU(2)_L \times U(1)_Y$ by boundary conditions [19]. In contrast to this, we suppose that $SU(5)$ is sponta-

neously broken to $G_{\rm SM}$ by a $\mathbf{24}^H$ bulk Higgs hypermultiplet propagating in the two throats that acquires a vacuum expectation value pointing in the hypercharge direction $\langle \mathbf{24}^H\rangle \propto \text{diag}(-\frac{1}{2},-\frac{1}{2},\frac{1}{3},\frac{1}{3},\frac{1}{3})$. Therefore, we have the usual $SU(5)$ explanation of SM quantum numbers and charge quantization.

The zero modes of all three generations of fermion superfields are assumed to be symmetrically localized at both IR branes of the throats. This symmetric trapping of zero modes at the tip of the throats could be achieved as in Ref. [17] (see also Ref. [20]) by introducing suitable bulk fermion masses in the throats. As a result of this localization, we describe the fermion zero modes in the language of 4D $N=1$ SUSY. In doing so, it is assumed that some 4D $N=2$ SUSY (which is equivalent to minimal 5D SUSY) is locally broken down to 4D $N=1$ SUSY at the UV/IR branes.

The quark and lepton zero modes are contained in the $SU(5)$ matter chiral superfields $\mathbf{10}_i$ and $\overline{\mathbf{5}}_i$, where $i=1,2,3$ is the generation index. To obtain small neutrino masses via the type-I seesaw mechanism [3], we introduce three right-handed $SU(5)$ singlet neutrino superfields $\mathbf{1}_i$. The 5D Lagrangian for the Yukawa couplings of the zero mode fermions then reads

$$\begin{aligned}\mathcal{L}_{5D} &= \int d^2\theta\big[\delta(y_1-\pi R_1)\big(\tilde{Y}^u_{ij,R_1}\mathbf{10}_i\mathbf{10}_j\mathbf{5}^H \\ &+ \tilde{Y}^d_{ij,R_1}\mathbf{10}_i\overline{\mathbf{5}}_j\overline{\mathbf{5}}^H + \tilde{Y}^\nu_{ij,R_1}\overline{\mathbf{5}}_i\mathbf{1}_j\mathbf{5}^H + M_R\tilde{Y}^R_{ij,R_1}\mathbf{1}_i\mathbf{1}_j\big) \\ &+ \delta(y_2-\pi R_2)\big(\tilde{Y}^u_{ij,R_2}\mathbf{10}_i\mathbf{10}_j\mathbf{5}^H + \tilde{Y}^d_{ij,R_2}\mathbf{10}_i\overline{\mathbf{5}}_j\overline{\mathbf{5}}^H \\ &+ \tilde{Y}^\nu_{ij,R_2}\overline{\mathbf{5}}_i\mathbf{1}_j\mathbf{5}^H + M_R\tilde{Y}^R_{ij,R_2}\mathbf{1}_i\mathbf{1}_j\big) + \text{h.c.}\big], \end{aligned} \tag{3}$$

where $\tilde{Y}^x_{ij,R_1}$ and $\tilde{Y}^x_{ij,R_2}$ $(x=u,d,\nu,R)$ are Yukawa coupling matrices (with mass dimension $-1/2$) and $M_R\simeq 10^{14}\,\text{GeV}$ is the $B-L$ breaking scale. In the four-dimensional (4D) low energy effective theory, $\mathcal{L}_{5D}$ gives rise to the 4D Yukawa couplings

$$\begin{aligned}\mathcal{L}_{4D} &= \int d^2\theta\big[Y^u_{ij}\mathbf{10}_i\mathbf{10}_j\mathbf{5}^H + Y^d_{ij}\mathbf{10}_i\overline{\mathbf{5}}_j\overline{\mathbf{5}}^H \\ &+ Y^\nu_{ij}\overline{\mathbf{5}}_i\mathbf{1}_j\mathbf{5}^H + M_R Y^R_{ij}\mathbf{1}_i\mathbf{1}_j + \text{h.c.}\big],\end{aligned} \tag{4}$$

where $Y^x_{ij}=(M_*\pi R)^{-1/2}(\tilde{Y}^x_{ij,R_1}+\tilde{Y}^x_{ij,R_2})$ are the dimensionless Yukawa coupling matrices of the low-energy theory, $M_*\simeq(M^2_{\rm Pl}R^{-1}_{1,2})^{1/3}$ is the fundamental scale, and $M_{\rm Pl}\simeq 10^{19}\,\text{GeV}$ the usual 4D Planck scale. Note from Eq. (4) that small neutrino masses are generated via the canonical type-I seesaw mechanism after integrating out the $\mathbf{1}$s. A crucial property of the 4D Yukawa couplings Y^x_{ij}, which we will exploit later, is that they receive contributions from both IR branes of the throats.

Now, we extend the gauge group to $SU(5)\times G_F$, where G_F is a discrete non-Abelian flavor symmetry group. We assume that G_F is a semi-direct product of two flavor groups G_A and G_B, i.e. $G_F=G_A\ltimes G_B$. Here, G_A is taken to be a direct product of Z_n symmetries, i.e.

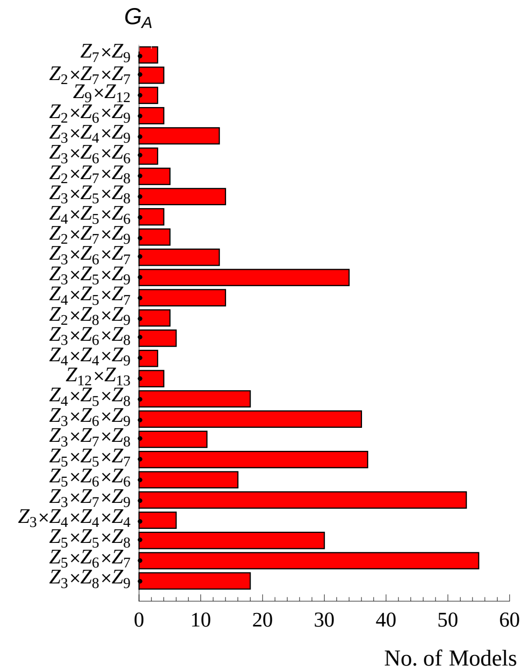

FIG. 2: Overview of $SU(5)\times G_A$ models for varying flavor group G_A. Each model yields an excellent fit to quark and lepton masses, $V_{\rm CKM}$, and nearly tribimaximal lepton mixing. All models produce a reactor neutrino angle $\ll 1^\circ$ and a normal neutrino mass hierarchy. The graph summarizes about 4×10^2 realistic GUTs.

$G_A=Z_{n_1}\times Z_{n_2}\times\cdots\times Z_{n_m}$, where m is the number of Z_n factors and the n_k $(k=1,2,\ldots,m)$ may be different. Under G_A, we assign to each generation i the charges

$$\begin{aligned}\mathbf{10}_i &\sim (p^i_1,p^i_2,\ldots,p^i_m),\\ \overline{\mathbf{5}}_i &\sim (q^i_1,q^i_2,\ldots,q^i_m),\\ \mathbf{1}_i &\sim (r^i_1,r^i_2,\ldots,r^i_m),\end{aligned} \tag{5}$$

where the jth entry in each row vector denotes the Z_{n_j} charge of the representation. In the 5D theory, we suppose that the group G_A is spontaneously broken by singly charged flavon fields located at the IR branes. The Yukawa coupling matrices of quarks and leptons are then generated by the Froggatt-Nielsen mechanism [21].

Applying a straightforward generalization of the flavor group space scan in Ref. [16] to the $SU(5)\times G_A$ representations in Eq. (5), we find a large number of about 4×10^2 flavor models that produce the hierarchies of quark and lepton masses and yield the CKM and PMNS mixing angles in perfect agreement with current data. A distribution of these models as a function of the group G_A for increasing group order is shown in FIG. 2. The selection criteria for the flavor models are as follows: First, all models have to be consistent with the quark and charged

lepton mass ratios

$$
\begin{aligned}
m_u : m_c : m_t &= \epsilon^6 : \epsilon^4 : 1, \\
m_d : m_s : m_b &= \epsilon^4 : \epsilon^2 : 1, \\
m_e : m_\mu : m_\tau &= \epsilon^4 : \epsilon^2 : 1,
\end{aligned} \tag{6}
$$

and a normal hierarchical neutrino mass spectrum

$$
m_1 : m_2 : m_3 = \epsilon^2 : \epsilon : 1, \tag{7}
$$

where $\epsilon \simeq \theta_\mathrm{C} \simeq 0.2$ is of the order of the Cabibbo angle. Second, each model has to reproduce the CKM angles

$$
V_{us} \sim \epsilon, \quad V_{cb} \sim \epsilon^2, \quad V_{ub} \sim \epsilon^3, \tag{8}
$$

as well as nearly tribimaximal lepton mixing at 3σ CL with an extremely small reactor angle $\lesssim 1^\circ$. In performing the group space scan, we have restricted ourselves to groups G_A with orders roughly up to $\lesssim 10^2$ and FIG. 2 shows only groups admitting more than three valid models. In FIG. 2, we can observe the general trend that with increasing group order the number of valid models per group generally increases too. This rough observation, however, is modified by a large "periodic" fluctuation of the number of models, which possibly singles out certain groups G_A as particularly interesting. The highly populated groups would deserve further systematic investigation, which is, however, beyond the scope of this paper.

From this large set of models, let us choose the group $G_A = Z_3 \times Z_8 \times Z_9$ and, in the notation of Eq. (5), the charge assignment

$$
\begin{aligned}
&\mathbf{10}_1 \sim (1,1,6),\ \ \mathbf{10}_2 \sim (0,3,1),\ \ \mathbf{10}_3 \sim (0,0,0), \\
&\overline{\mathbf{5}}_1 \sim (1,4,2),\ \ \overline{\mathbf{5}}_2 \sim (0,7,0),\ \ \overline{\mathbf{5}}_3 \sim (0,0,1), \\
&\mathbf{1}_1 \sim (2,0,6),\ \ \mathbf{1}_2 \sim (2,6,0),\ \ \mathbf{1}_3 \sim (2,0,6),
\end{aligned} \tag{9}
$$

as a showcase. The Froggatt-Nielsen mechanism then generates the Yukawa coupling textures (neglecting $\mathcal{O}(1)$ coefficients)

$$
Y^u_{ij} \sim \begin{pmatrix} \epsilon^6 & \epsilon^7 & \epsilon^5 \\ \epsilon^7 & \epsilon^4 & \epsilon^4 \\ \epsilon^5 & \epsilon^4 & 1 \end{pmatrix},\ Y^d_{ij} \sim \epsilon \begin{pmatrix} \epsilon^4 & \epsilon^3 & \epsilon^3 \\ \epsilon^4 & \epsilon^2 & \epsilon^4 \\ \epsilon^6 & 1 & 1 \end{pmatrix}, \tag{10}
$$

$$
Y^\nu_{ij} \sim \epsilon^3 \begin{pmatrix} \epsilon^2 & \epsilon & \epsilon^2 \\ \epsilon^2 & \epsilon & \epsilon^2 \\ 1 & \epsilon & 1 \end{pmatrix},\ Y^R_{ij} \sim \epsilon^4 \begin{pmatrix} 1 & \epsilon^2 & 1 \\ \epsilon^2 & \epsilon & \epsilon^2 \\ 1 & \epsilon^2 & 1 \end{pmatrix}, \tag{11}
$$

from higher-dimension operators. Observe that G_A produces overall suppression factors in front of the down quark and neutrino Yukawa coupling matrices. Only the top Yukawa coupling is not suppressed by the flavor symmetry. Note that as long as $M_* R_{1,2} \lesssim 16\pi^2$, the top Yukawa coupling can be large without requiring strong coupling below M_* [22]. In our example, we have a moderate $\tan\beta \sim 10$, and since $SU(5)$ is preserved on both throats, the charged lepton Yukawa coupling matrix Y^e_{ij} satisfies $Y^e_{ij} = Y^d_{ji}$. Thus, the model exhibits $b-\tau$ unification and the usual $SU(5)$ mass relations for the first two generations. From Eqs. (10) and (11), we see that the model predicts the quark and lepton mass ratios in Eqs. (6) and (7). Realistic relations between the first two generations of charged fermion masses may then be achieved by the Georgi-Jarlskog mechanism [23]. For the quarks, the model predicts the CKM angles in Eq. (8). The leptonic sector corresponds (up to rotations of the **10**s) to No. 64 in the list of 1981 matrix sets in Ref. [15]. Hence, the $\mathcal{O}(1)$ Yukawa coupling coefficients can be fitted such that the PMNS mixing angles are in perfect agreement with current neutrino data at 1σ CL. As a result, in U_PMNS, the solar, atmospheric, and reactor angle, take the values $\theta_{12} \approx 34^\circ, \theta_{23} \approx 52^\circ$, and $\theta_{13} \approx 0.2^\circ$, respectively. The PMNS mixing angles describe therefore nearly tribimaximal lepton mixing with an extremely small reactor angle $\theta_{13} \ll 1^\circ$. At the same time, the heavy right-handed neutrino masses m^R_i exhibit the hierarchical mass ratios $m^R_1 : m^R_2 : m^R_3 = \epsilon^2 : \epsilon : 1$. We suppose that the flavor symmetries are broken at high energies such as the GUT scale. The Cabibbo angle θ_C, however, is practically stable under renormalization group running and $V_{cb} \sim \epsilon^2$ changes by a factor less than 2 when running from the Planck scale down to low energies [24]. In the neutrino sector, since the light neutrinos have a normal hierarchical mass spectrum, renormalization group effects have hardly any impact on the mass ratios in Eq. (7) and alter the PMNS mixing angles only by $\ll 1^\circ$ (for a more detailed discussion and references see Ref. [15]). Within our precision, we will therefore neglect the modification of our predictions by renormalization group effects.

While $G_A \subset G_F$ controls the order of magnitude of the Yukawa couplings, exact relations among the Yukawa coupling matrix elements are established by $G_B \subset G_F$. We suppose that $G_B = G_{B_1} \times G_{B_2} \times G_{B_3}$ is a direct product of discrete groups which act (up to conjugation by G_A) on the multiplets as

$$
G_{B_1} : \overline{\mathbf{5}}_2 \leftrightarrow \overline{\mathbf{5}}_3, \quad G_{B_2} : \mathbf{1}_1 \leftrightarrow \mathbf{1}_3, \quad G_{B_3} : \mathbf{10}_3 \rightarrow -\mathbf{10}_3.
$$

Since the permutation symmetry G_{B_1} does not commute with G_A (whereas G_{B_2} and G_{B_3} commute with G_A), the total discrete flavor symmetry group G_F is non-Abelian and given by a semi-direct product $G_F = G_A \ltimes G_B$. The symmetries G_{B_1} and G_{B_2} establish the exact relations

$$
Y^d_{32} = Y^d_{33}, \quad Y^\nu_{21} = Y^\nu_{23}, \quad Y^\nu_{31} = Y^\nu_{33}, \quad Y^R_{11} = Y^R_{33} \tag{12}
$$

for the Yukawa couplings in Eq. (4). To avoid wrong predictions for other Yukawa couplings, G_B has to be broken. Recall from Eq. (3) that the 4D Yukawa couplings receive contributions from both IR branes of the throats. We therefore assume that G_B is locally broken

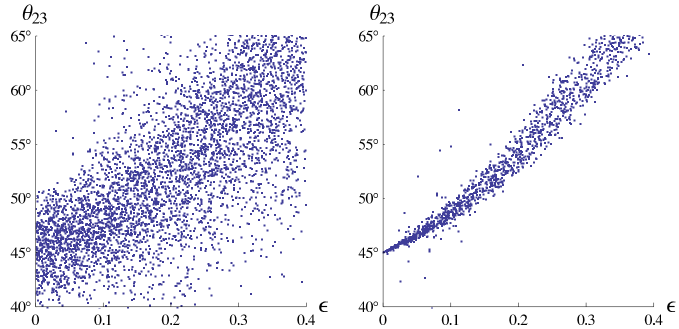

FIG. 3: Effect of the non-Abelian flavor symmetry on θ_{23} for a 10% variation of all Yukawa couplings. Shown is θ_{23} as a function of ϵ for the flavor group G_A (left) and $G_A \ltimes G_B$ (right). The right plot illustrates the exact prediction of the zeroth order term $\pi/4$ in the expansion $\theta_{23} = \pi/4 + \epsilon/\sqrt{2}$ due to the non-Abelian nature of the flavor symmetry.

at the IR branes as follows:

$$\begin{aligned} \text{at } y_1 = \pi R_1 &\;:\; G_B \to G_{B_1}, \\ \text{at } y_2 = \pi R_2 &\;:\; G_B \to G_{B_2} \times G_{B_3}. \end{aligned} \tag{13}$$

Taking the total contribution to the Yukawa coupling matrices that are generated by G_F at both IR branes into account, we reproduce the textures in Eqs. (10) and (11). Now, however, the textures obey the additional exact relations in Eq. (12).

A Monte Carlo scan of the $\mathcal{O}(1)$ Yukawa coupling coefficients (cf. Ref. [16]) then shows that the model satisfies the sum rule $\theta_{23} = \pi/4 + \epsilon/\sqrt{2}$ and the relation $\theta_{13} \simeq \epsilon^2$. The important point is that in the expression for θ_{23}, the leading order term $\pi/4$ is exactly predicted by the non-Abelian flavor symmetry $G_F = G_A \ltimes G_B$ (see FIG. 3), while $\theta_{13} \simeq \theta_{\mathrm{C}}^2$ is extremely small due to a suppression by the square of the Cabibbo angle. We thus predict a deviation $\sim \epsilon/\sqrt{2}$ from maximal atmospheric mixing, which can be tested in future neutrino oscillation experiments such as NOνA, T2K, or a neutrino factory [25]. The scan also shows that the model can, at the same time, accommodate the sum rule $\theta_{12} \approx \pi/4 - \epsilon/\sqrt{2}$, which is the well-known QLC relation for the solar angle. There have been attempts in the literature to reproduce QLC in quark-lepton unified models [26], however, the model presented here is the first realization of QLC in an $SU(5)$ GUT. Although our analysis has been carried out for the CP conserving case, a simple numerical study shows that CP violating phases (cf. Ref. [27]) relevant for neutrinoless double beta decay and leptogenesis can be easily included as well.

Concerning proton decay, note that since $SU(5)$ is broken by a bulk Higgs field, the broken gauge boson masses are $\simeq M_{\mathrm{GUT}}$. Therefore, all fermion zero modes can be localized at the IR branes of the throats without introducing rapid proton decay through $d = 6$ operators. To achieve doublet-triplet splitting and suppress $d = 5$ proton decay, we may then, e.g., resort to suitable extensions of the Higgs sector [28]. Moreover, although the flavor symmetry G_F is global, quantum gravity effects might require G_F to be gauged [29]. Anomalies can then be canceled by Chern-Simons terms in the 5D bulk.

We emphasize that the above discussion is focussed on a specific minimal example realization of the model. Many $SU(5)$ GUTs with non-Abelian flavor symmetries, however, can be constructed along the same lines by varying the flavor charge assignment, choosing different groups G_F, or by modifying the throat geometry. A detailed analysis of these models and variations thereof will be presented in a future publication [30].

To summarize, we have discussed the construction of 5D SUSY $SU(5)$ GUTs that yield nearly tribimaximal lepton mixing, as well as the observed CKM mixing matrix, together with the hierarchy of quark and lepton masses. Small neutrino masses are generated only by the type-I seesaw mechanism. The fermion masses and mixings arise from the local breaking of non-Abelian flavor symmetries at the IR branes of a flat multi-throat geometry. For an example realization, we have shown that the non-Abelian flavor symmetries can exactly predict the leading order term $\pi/4$ in the sum rule for the atmospheric mixing angle, while strongly suppressing the reactor angle. This makes this class of models testable in future neutrino oscillation experiments. In addition, we arrive, for the first time, at a combined description of QLC and non-Abelian flavor symmetries in $SU(5)$ GUTs. One main advantage of our setup with throats is that the necessary symmetry breaking can be realized with a very simple Higgs sector and that it can be applied to and generalized for a large class of unified models.

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